2019

Unique Paper Code	:	235301 (21)
Name of the Paper	:	MAHT 301-Calculus- II
Name of the Course	:	B.Sc. (Hons.) Mathematics- II
Semester	:	III
Duration	:	3 hours
Maximum Marks	:	75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory.
3. Attempt any **five** questions from each Section.
4. **All** questions carry equal marks.

SECTION - I

1. (a) Let f be the function defined by :

$$f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for, } (x,y) = (0,0) \end{cases}$$

Is f continuous at $(0,0)$? Explain.

(b) The **Cauchy-Riemann equations** are

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

where $u = u(x,y)$ and $v = v(x,y)$. Check if $u = e^{-x}cosy,\ v = e^{-x}siny$ satisfy the Cauchy-Riemann equations?

2. In physics, the *wave equation* is

$$\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{\partial x^2}$$

and the *heat equation* is

$$\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2}$$

Determine whether $z = e^{-1}\left(\sin\frac{x}{c} + \cos\frac{x}{c}\right)$ satisfies the wave equation, the heat equation, or neither.

3. An open box has length 3ft, width 1ft, and height 2 ft and is constructed from material that costs \$2/ft^2 for the sides and \$3/ft^2 for the bottom. Compute the cost of constructing the box, and then use increments to estimate the change in cost if the length and width are each increased by 3 in. and the height is decreased by 4 in.

4. If $z = u + f(uv)$, show that

$$u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v} = u$$

5. (a) Find the directional derivative of $f(x,y) = ln(x^2 + y^3)$ at $P_0(1,-3)$ in the direction of $v = 2\mathbf{i} - 3\mathbf{j}$.

(b) Sketch the level curve corresponding to $C = 1$ for the function $f(x,y) = x^2 - y^2$ and find a normal vector at the point $P_0(2,\sqrt{3})$.

6. Find the maximum and minimum values of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ over the triangle with vertices (0,0), (9,0) and (0,9).

SECTION-II

7. Evaluate $\iint \frac{dA}{1+y^2}$ over a triangle D bounded by $x = 2y, y = -x$ and $y = 2$.

8. Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9-x^2-y^2}}\, dxdy$ by converting to polar coordinates.

9. Find the volume V of the tetrahedron T bounded by the plane $x + y + z = 1$ and coordinate planes $x = 0, y = 0$ and $z = 0$.

10. Use spherical coordinates to evaluate $\iiint_D \frac{dxdydz}{\sqrt{x^2+y^2+z^2}}$ where D is the region given by $x^2 + y^2 + z^2 \leq 3, z \geq 0$.

11.Compute the integral $\iiint (x^4 + 2x^2y^2 + y^4)dxdydz$ over the cylindrical solid $x^2 + y^2 \leq a^2$ with $0 \leq z \leq \frac{1}{\pi}$.

12. Use suitable change of variables to find the area of the region R bounded by the hyperbolas $xy = 1$ and $xy = 4$ and lines $y = x$ and $y = 4x$.

SECTION-III

13. A force field in the plane is given by $\mathbf{F} = (x^2 - y^2)\mathbf{i} + 2xy\mathbf{j}$. Find the total work done by this force in moving a point mass counterclockwise around the square with vertices (0,0), (2,0), (2,2), (0,2).

14.(a) Show that the force field $\mathbf{F} = \sin\, z\,\mathbf{i} - z\, sin\, y\mathbf{j} + (xcos\, z + \cos y)\mathbf{k}$ is conservative.

(b) Verify that $\int_C [(3x^2 + 2x + y^2)dx + (2xy + y^3)dy]$, where C is any path from (0,0) to (1,1), is independent of path.

15. Use Green's theorem to find the work done by the force field

$$F(x,y) = (x + 2y^2)\mathbf{j}$$

as the object moves once counterclockwise about the circle $(x - 2)^2 + y^2 = 1$.

16. Evaluate surface integral $\iint_S \sqrt{1 + 4z}\; dS$ where S is the portion of the paraboloid $z = x^2 + y^2$ for which $z \leq 4$.

17.Use Stokes' Theorem to evaluate $\oint_C \mathbf{F}.d\mathbf{R}$ where $\mathbf{F} = 2y\mathbf{i} - 6z\mathbf{j} + 3x\mathbf{k}$ and C is the intersection of the xy-plane and paraboloid $z = 4 - x^2 - y^2$, traversed counter clockwise as viewed from above.

18.Use the divergence theorem to evaluate $\iint_S \mathbf{F}.\mathbf{N}dS$ where $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + x^3y^3\mathbf{k}$ and S is the tetrahedron bounded by the plane $x + y + z = 1$ and the coordinate planes with outward unit normal vector $\mathbf{N}$.